Vagueness & Polysemy 2

Semantic Approaches to Vagueness: S'valuationism, Contextualism, and Epistemicism

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Main questions for the course

Main Questions

- Why is vagueness a challenge for a formal semantics for natural languages?
  - Day 1: Background
    - Properties of Classical First Order Logic (CFOL) & connection to modern semantic theory
    - Diagnosing Vagueness
- **What are the main reactions to this challenge?**
  - Days 1-3: Responses to vagueness
    - fuzziness (day 1)
    - **S'valuationism & polysemy, contextualism, degrees (day 2)**
    - probabilistic approaches (day 3)
- How does vagueness connect to polysemy?
  - **Part of Day 2: Is vagueness a kind of polysemy?**
  - Day 4: The common origins of vagueness and polysemy?

Plan for Day 2

1. S'valuationism: Vagueness as a kind of polysemy
   - Supervaluationism: truth on all precisifications
   - Subvaluationism: truth on any precisifications
   - Challenges for S'valuationism
   - S'valuationism and polysemy
2. Contextualism: Shifting contexts
3. Epistemicism: Relocating the problem
   - Epistemicism and degree semantics

What is S'valuationism?

Not really one theory at all but two:

- Supervaluationism
- Subvaluationism

Term coined by Ripley (2013) or Cobreros et al. (2011)

Fine (1975, p. 282) on supervaluationism (but it could apply to S'valuation more generally)

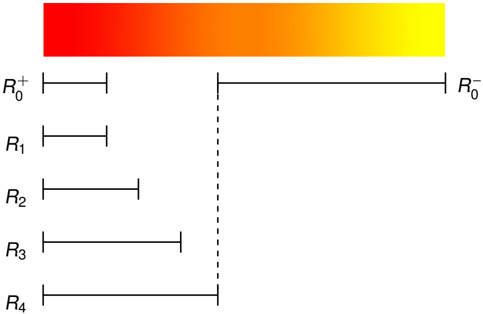
- "Vagueness is ambiguity on a grand and systematic scale."
  - But, as we'll see, if anything, on S'valuationism, it is closer to polysemy.

S'valuationism starts from the assumptions that

- vague expressions have numerous admissible precise interpretations
- truth evaluations can be defined in terms of the classical evaluations of all of the admissible precisifications

Basic idea: Admissible precisifications

An expression like *red* is multiply polysemous



- The base meaning of *red*, R_0 is a partial function on the domain
  - Leaves a gap
- Precisifications R_1, R_2, R_4, R_4
  - Extend the base extension
  - form a partial order $R_1 \propto R_2 \propto R_3 \propto R_4$
    - ★ $A \propto B$ = B precisifies A
  - All subsume R_0^+
  - Do not overlap with R_0^-

Supervaluation vs. Subvaluation

Supervaluations

For a predicate P_0, $P_0(a)$ is true iff for **all** P_i that are admissible precisifiactions of P_0, $P_i(a)$ is true.

Subvaluations

For a predicate P_0, $P_0(a)$ is true iff for **some** P_i that is an admissible precisifiactions of P_0, $P_i(a)$ is true.

Supervaluationism: Don't worry, be gappy

- Pioneered as an approach to vagueness by Mehlberg (1958)
- Brought to more prominent attention by Fine (1975), Kamp (1975) and Kamp and Partee (1995)[1]

Supervaluationism is gappy

For a predicate P_0, $P_0(a)$ is true iff for **all** P_i that are admissible precisifications of P_0, $P_i(a)$ is true.

- a proposition is true iff it is true on all valuations
- a proposition is false iff it is false on all valuations
- a proposition is neither true nor false if it is not true and not false
  - Gaps in supervaluations even if no precisification has any gaps
- an advance on merely "gappy" accounts since one can supervaluate complex propositions as well as atomic ones.

[1] There is an outline of a degree based form of supervaluationism in Lewis (1970).

Supervaluationism and classical theorems

All classical theorems are valid on a supervaluational approach

- however the consequence relation differs (more on this later).

For example:

- Instances of excluded middle are all true because they are true on all classical precisifications ($\vDash_{\text{superval}} \phi \vee \neg\phi$)
- Instances of non-contradiction are all false because they are false on all classical precisifications ($\vDash_{\text{superval}} \neg(\phi \wedge \neg\phi)$).

Assume that $P_0^+ = \{a, b\}$ and $P_0^- = \{f, g\}$. Furthermore that:

$$P_1 = \{a, b\} \qquad P_2 = \{a, b, c\}$$
$$P_3 = \{a, b, c, d\} \qquad P_4 = \{a, b, c, d, e\}$$

Since P_1, P_2,P_3, and P_4 are all classical, total partitions, at every precisification, P_i, and for every member $x \in D = \{a, b, c, d, e\}$:

- it is true that $P_i(x) \vee P_i(x)$
- it is true that $\neg(P_i(x) \wedge \neg P_i(x))$

Hence these are supervaluationally valid.

Supervaluationism and Bivalence

The metasemantic Principe of Bivalence does not hold in a supervaluationist system

- a proposition can be neither true nor false
  - i.e. true on some admissible precisifications and false on others.

Take our mini model from before:

$$P_1 = \{a, b\} \qquad P_2 = \{a, b, c\}$$
$$P_3 = \{a, b, c, d\} \qquad P_4 = \{a, b, c, d, e\}$$

Is it true that $P_0(c)$?

- No: $P_i(c)$ is not true on all precisifications P_i

But it is also not false:

- $P_i(c)$ is not false on all precisifications P_i i.e. true on some admissible precisifications and false on others.

Weak paracompleteness I

Classical logic does not distinguish between the following consequences.

- For any proposition ψ, one can conclude the multiple conclusion $\phi, \neg\phi$ (at least one of ϕ and $\neg\phi$ is entailed), or that $\phi \vee \neg\phi$ is true.

$$\psi \vDash_{\mathrm{CL}} \phi \vee \neg\phi$$
$$\psi \vDash_{\mathrm{CL}} \phi, \neg\phi$$

Semantic models based on CL have only one valuation (a classical one). Hence, for any valuation in which $\phi \vee \neg\phi$ holds, the same valuation will mean that either ϕ or $\neg\phi$ holds.

Supervaluationism is weakly paracomplete (Hyde 2008)

In contrast, supervaluationist consequence supports one but not the other:

$$\psi \vDash_{\mathrm{superval}} \phi \vee \neg\phi$$
$$\psi \nvDash_{\mathrm{superval}} \phi, \neg\phi$$

Weak paracompleteness II

Supervaluationism is weakly paracomplete (Hyde 2008)

In contrast, supervaluationist consequence supports one but not the other:

$$\psi \vDash_{\text{superval}} \phi \vee \neg\phi$$
$$\psi \nvDash_{\text{superval}} \phi, \neg\phi$$

The reason for this is:

- on every classical valuation, it must be true true that $\phi \vee \neg\phi$
- however, it is possible that neither ϕ nor $\neg\phi$ are true across all valuations
- hence one cannot conclude that one of ϕ and $\neg\phi$ are true.

Weak paracompleteness III

Supervaluationism is weakly paracomplete (Hyde 2008)

In contrast, supervaluationist consequence supports one but not the other:

$$\psi \vDash_{\text{superval}} \phi \vee \neg\phi$$
$$\psi \nvDash_{\text{superval}} \phi, \neg\phi$$

An example of such an inference from Edgington (1997, p. 310)

- "A library book can be such that it is not clear whether it should be classified as Philosophy of Language or Philosophy of Logic; but if we have a joint category for books of either kind, it clearly belongs there."

There is, however, some debate about whether such examples are persuasive (Hyde 2008, ch. 4).

Supervaluationism and the sorites

Assume that $Q_0^+ = \{a_1, a_2\}$ and that $Q_0^+ = \{a_5, a_6\}$
There are three precisifications of Q_0:

- $Q_1 = \{a_1, a_2\}$
- $Q_2 = \{a_1, a_2, a_3\}$
- $Q_3 = \{a_1, a_2, a_3, a_4\}$

A simple sorites argument:

(P1) $Q_0(a_1)$

(P2) $\neg Q_0(a_6)$

(P3) $Q_0(a_1) \rightarrow Q_0(a_2)$

(P4) $Q_0(a_2) \rightarrow Q_0(a_3)$

(P5) $Q_0(a_3) \rightarrow Q_0(a_4)$

(P6) $Q_0(a_4) \rightarrow Q_0(a_5)$

(P7) $Q_0(a_5) \rightarrow Q_0(a_6)$

(C) $Q_0(a_6) \wedge \neg Q_0(a_6)$

- (P1), (P2), (P3), and (P7) supervaluate as true
- (C) supervaluates as false
- (P4) is false for Q_1 and true for Q_2 and Q_3
- (P5) is true for Q_1, false for Q_2 and true for Q_3
- (P6) is true for Q_1, and Q_2 and false Q_3
  - So (P4), (P5) and (P6) are neither true nor false

The argument is valid (but unsound, since not all of its premises are true).

Supervaluationism and the sorites

However, something slightly strange happens for the short sorites:
Again, assume that $Q_0^+ = \{a_1, a_2\}$ and that $Q_0^- = \{a_5, a_6\}$ and that there are three precisifications of Q_0:

- $Q_1 = \{a_1, a_2\}$
- $Q_2 = \{a_1, a_2, a_3\}$
- $Q_3 = \{a_1, a_2, a_3, a_4\}$

A short sorites argument:

(P1) $Q_0(a_1)$

(P2) $\neg Q_0(a_6)$

(P3) $\forall x_i \forall x_{i+1}[Q_0(x_i) \rightarrow Q_0(x_{i+1})]$

(C) $Q_0(a_6) \wedge \neg Q_0(a_6)$

- (P1), and (P2) supervaluate as true
- (P3) and (C) supervaluate as false

The argument is still valid (but unsound, since P3 is false).

However:

- If $\forall x_i \forall x_{i+1}[Q_0(x_i) \rightarrow Q_0(x_{i+1})]$ supervaluates as false, then $\exists x_i \exists x_{i+1}[Q_0(x_i) \wedge \neg Q_0(x_{i+1})]$ supervaluates as true

But that means that Q_0 has sharp boundaries. Is it still vague?

- Sharp boundaries, but it is indeterminate where they are.

Criticisms of supervaluationism

Predominantly on the failure of tolerance that we have just seen.

- Statements of tolerance e.g., $\forall x \forall y[(P(x) \wedge x \sim_P y) \rightarrow P(y)]$ come out as false for any P that is not a total function.
  - See Hyde (2008, ch. 4) for a review of some responses
- We are asked to distinguish two claims:
  - (i) It is true that there is some a and b such that $a \sim_P b$, and $P(a)$ and not-$P(b)$
  - (ii) There is some a and b such that it is true that $a \sim_P b$, and $P(a)$ and not-$P(b)$
- Not so clear that there is the requisite reading of anti-tolerance statements

Criticisms of supervaluationism

Another puzzle comes from paraconsistancy:

- One can always truly assert an instance of excluded middle, since all instances are supervaluationally true.
- But the multipremise conclusion: *either ϕ is true or $\neg\phi$ is true* cannot always be truly asserted.

Recall the kind of justification for this:

"A library book can be such that it is not clear whether it should be classified as Philosophy of Language or Philosophy of Logic; but if we have a joint category for books of either kind, it clearly belongs there."

- But: "If one means by 'a joint category for books of either kind' a category that includes all those in the category Philosophy of Language and all those in the category Philosophy of Logic then the book in question is not clearly in this simple disjunctive category." Hyde (2008, p. 87)

Subvaluationism: To be *and* not to be

- defended, for example, in Hyde (1997), and more recently in Hyde and Colyvan (2008); Cobreros (2011)
- until recently received less attention that its supervaluational sister

Subvaluationism is glutty

For a predicate P_0, $P_0(a)$ is true iff for **some** P_i that is an admissible precisification of P_0, $P_i(a)$ is true.

- a proposition is true iff it is true on some valuation
- a proposition is false iff it is false on some valuation
- a proposition is neither true nor false if it is not true and not false
  - There are gluts in subvaluations even if no precisification has any gluts
- like supervaluationism, one can subvaluate complex propositions as well as atomic ones.

Subvaluationism is the dual of supervaluationism

- $T(\phi)$ iff $\forall p[\phi \propto p \wedge T(p)]$ supervaluationism
- $T(\phi)$ iff $\exists p[\phi \propto p \wedge T(p)]$ subvaluationism

Subvaluationism and no gluts

No gluts does not hold for subvaluationism

No Gluts:

For all $\mathcal{I}$, all $a \in D$ and all predicates P,
It is not the case that $\mathcal{I}(P(a)) = 1$ and $\mathcal{I}(P(a)) = 0$.

- So, the subvaluational truth 'function' is not properly speaking a function at all
  - it assigns more than one value to some propositions.

Subvaluationism and classical theorems

All classical theorems are valid on a subvaluational approach

- the multi-premise consequence relation differs (more on this later).

For example:

- Instances of excluded middle are all true because they not false on any classical precisifications ($\vDash_{\text{subval}} \phi \vee \neg\phi$)
- Instances of contradiction are all false because they are not true on any classical precisifications ($\vDash_{\text{subval}} \neg(\phi \wedge \neg\phi)$).

Assume that $P_0^+ = \{a, b\}$ and $P_0^- = \{f, g\}$. Furthermore that:

$$\begin{array}{ll} P_1 = \{a, b\} & P_2 = \{a, b, c\} \\ P_3 = \{a, b, c, d\} & P_4 = \{a, b, c, d, e\} \end{array}$$

Since P_1, P_2,P_3, and P_4 are all classical, total partitions, at every precisification, P_i, and for every member $x \in D = \{a, b, c, d, e\}$:

- it is not false that $P_i(x) \vee P_i(x)$
- it is not false that $\neg(P_i(x) \wedge \neg P_i(x))$

Hence these are subvaluationally valid.

Subvaluationism is weakly paraconsistant I

Although Non-Contradiction holds ($\vDash_{\text{subval}} \neg(\phi \wedge \neg\phi)$), the semantic equivalent of non-contradiction fails.

- it is not true on subvaluationism that no proposition is true and false.

The extent to which subvaluationism is paraconsistant is constrained.

- Supervaluationism was weakly paracomplete
- Subvaluationism is *weakly paraconsistant*

In FCOL, both a single premise contradiction and a set of inconsistent premises lead to explosion:

Contradiction with Explosion:

For all formulas ϕ, ψ,

$$\{\phi, \neg\phi\} \vDash_{CL} \psi$$
$$\phi \wedge \neg\phi \vDash_{CL} \psi$$

Subvaluationism is weakly paraconsistant II

However, subvaluationism distinguishes these two:

- the assumption of a contradiction $\phi \wedge \neg\phi$ leads to explosion
- a classically inconsistent set of premises e.g., $\{\phi, \neg\phi\}$, does not

Weak paraconsistancy

$$\phi \wedge \neg\phi \vDash_{\text{subval}} \psi$$
$$\phi, \neg\phi \nvDash_{\text{subval}} \psi$$

On all classical precisifications, every statement of the form $\phi \wedge \neg\phi$ is false

- So for no statement of the form $\phi \wedge \neg\phi$ is it the case that some classical precisifications are true and others false.

However, for some statement ϕ, it may be the case that ϕ is true on some precisifications, but false on others

- So ϕ can be both subvaluationally true and subvaluationally false.

Subvaluationism is weakly paraconsistant III

A prima facie benefit of adopting a subvaluationist logic:

- it captures some of the empirical data that supervaluationism cannot
- for borderline cases of applying vague predicates, many people say *that is both P and not-P*

The classical and subvaluationist consequence relations diverge with respect to conjunction introduction:

$$\phi, \psi \vDash_{\mathrm{CL}} \phi \wedge \psi$$
$$\phi, \psi \nvDash_{\mathrm{subval}} \phi \wedge \psi$$

So there is an anomaly:

- So one can say *it is true that Alex is tall*
- and *it is true that she is not tall*
- but one cannot say *Alex is tall and not tall*

So we need a kind of *doublethink* when describing borderline cases
(or deny that "a is F and not F" expresses the proposition $F(a) \wedge \neg F(a)$).

Subvaluationism and the sorites I

Assume that $Q_0^+ = \{a_1, a_2\}$ and that $Q_0^+ = \{a_5, a_6\}$
There are three precisifications of Q_0:

- $Q_1 = \{a_1, a_2\}$
- $Q_2 = \{a_1, a_2, a_3\}$
- $Q_3 = \{a_1, a_2, a_3, a_4\}$

A short sorites argument:

(P1) $Q_0(a_1)$

(P2) $\neg Q_0(a_6)$

(P3) $Q_0(a_1) \rightarrow Q_0(a_2)$

(P4) $Q_0(a_2) \rightarrow Q_0(a_3)$

(P5) $Q_0(a_3) \rightarrow Q_0(a_4)$

(P6) $Q_0(a_4) \rightarrow Q_0(a_5)$

(P7) $Q_0(a_5) \rightarrow Q_0(a_6)$

(C) $Q_0(a_6) \wedge \neg Q_0(a_6)$

- (P1), (P2), (P3), and (P7) subvaluate as true
- (C) subvaluates as false
- (P4) is false for Q_1 and true for Q_2 and Q_3
- (P5) is true for Q_1, false for Q_2 and true for Q_3
- (P6) is true for Q_1, and Q_2 and false Q_3
  - So (P4), (P5) and (P6) are both true and false

Subvaluationism and the sorites II

Is the argument is valid?

- Unlike supervaluationism, we cannot say that it is valid, but unsound, since it is possible for all the premises to be true and for the conclusion to be true (propositions can be true and false)
- instead, it is not considered valid

Subvaluationism and modus ponens

It is not considered valid because modus ponens fails on subvaluationism:

Modus Ponens:

For all formulas ϕ, ψ,
$\{\phi \rightarrow \psi, \phi\} \vDash \psi$

- $Q_1 = \{a_1, a_2\}$
- $Q_2 = \{a_1, a_2, a_3\}$
- $Q_3 = \{a_1, a_2, a_3, a_4\}$

So (following the reasoning in Hyde (2008))

- $Q_0(a_4)$ is true and false, so it is true
- $Q_0(a_5)$ is determinately false
- $Q_0(a_4) \rightarrow Q_0(a_5)$ is true (because $Q_0(a_4)$ is false (as well as true))

So the premises in

$$Q_0(a_4) \rightarrow Q_0(a_5), Q_0(a_4) \vDash Q_0(a_5)$$

Are true, but the conclusion is false.

Subvaluationism and the short sorites

Again, there are problems for the short sorites:
Again, assume that $Q_0^+ = \{a_1, a_2\}$ and that $Q_0^+ = \{a_5, a_6\}$ and that there are three precisifications of Q_0:

- $Q_1 = \{a_1, a_2\}$
- $Q_2 = \{a_1, a_2, a_3\}$
- $Q_3 = \{a_1, a_2, a_3, a_4\}$

A short sorites argument:

(P1) $Q_0(a_1)$

(P2) $\neg Q_0(a_6)$

(P3) $\forall x_i \forall x_{i+1}[Q_0(x_i) \rightarrow Q_0(x_{i+1})]$

(C) $Q_0(a_6)$

- (P1), and (P2) subvaluate as true
- (P3) and (C) subvaluate as false

The short argument is valid, even though the long argument was not.

However, like with supervaluationism:

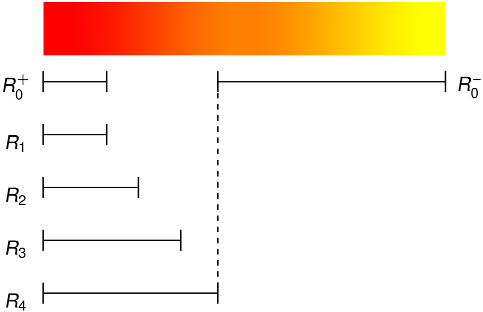
- If $\forall x_i \forall x_{i+1}[Q_0(x_i) \rightarrow Q_0(x_{i+1})]$ subvaluates as (determinately) false, then $\exists x_i \exists x_{i+1}[Q_0(x_i) \wedge \neg Q_0(x_{i+1})]$ subvaluates as (determinately) true

But that still means that Q_0 has sharp boundaries

Criticisms of Subvaluationism

- A cultural-historical problem?
  - the Russell-Carnap-Quine tradition which *de facto*, even if not *de jure*, makes a paraconsitant position such as Subvaluationism harder to convince people of.
- The failure of e.g. modus ponens and conjunction introduction.

Higher-Order Vagueness



Both forms of S'valuationism face the challenge of higher-order vagueness.

- Predicates on supervaluationism have extensions structured as True–Neither true nor false–False
- Predicates on subvaluationism have extensions structured as True–Both true and false–False

Higher-Order Vagueness II

In both cases, this means a sharp jump between *definitely true* and something else.

A possible fix:

- Introduce degrees
  - We'll look at what is essentially a probabilistic form of supervaluationism tomorrow.

- Nb. Subvaluationism has been argued to outperform both supervaluationism and classicism (Cobreros 2011).

S'valuationist logic and polysemy I

If vague expressions really are just underspecified polysemous expressions, we would expect the logic for them to work the same. But is this the case? – Suppose my new car is green in colour.

- *My new car is green*
  - My new car is green in colour
  - My new car is environmentally friendly
  - My new car is envious
  - My new car is inexperienced
- Is the sentence be neither true nor false? (supervaluationism)
  - Maybe. The *sentence* is neither true nor false (outside of context).
  - When uttered, the utterance expresses some more determinate sense
- Is the sentence be both true and false? (subvaluationism)
  - Maybe. The *sentence* is both true and false (outside of context).
  - When uttered, the utterance expresses some more determinate sense

S’valuationist logic and polysemy II

A test for polysemy (not universally applicable): Zeugma

(1) This product is suitable for vegetarians and home freezing.

- Coordinated reduced conjunction sentences are zeugmatic if more than one sense is required for interpretation

Good news for S’ valuationism?

(2) Mary had a little lamb and a small farmstead.

(3) Mary had a little lamb and some roast potatoes.

Recall:

- $\phi \wedge \psi$ is evaluated *as a whole* at each precisification
  - Prediction: It should be weird to mix senses across connectives if the logic for polysemy is S’valuational

S'valuationist logic and polysemy III

Comparing supervaluationism with subvaluationism:

(4) Mary had a little lamb and some carrots.

(a) Mary owned a little lamb and some carrots

(b) Mary ate a little lamb and some carrots

Supervaluationism

- True if (a) and (b) are true
- False if (a) and (b) are false
- Neither true nor false if only one of (a)/(b) is true

Supervaluationism

- True if one (a) or (b) is true
- False if one of (a) or (b) is false

S'valuationist logic and polysemy IV

A problem with enforcing a precisification over conjunctions:

(5) Alex ran the marathon

(a) Alex completed the marathon by running in it.
(b) Alex organised the marathon

The following is ok:

(6) Alex ran the marathon, but she didn't *run* the marathon

- Not clear how this can be accounted for with an S'valuationist logic

Is vagueness really polysemy?

A more general worry

- Vague expressions are often polysemous as well
  - The kind of way in which *green* is vague is not clearly the same as the way in which *green is polysemous*

Is vagueness really polysemy? II

A better connection between polysemy and vagueness?

- Languages could have:
  (a) Different expressions for every sense of a polysemous word
  (b) Different expressions for e.g. every degree of height or every distinguishable shade of colour
- If they did, communication would be easier (no vagueness or polysemy resolution)
- But universally languages are not like that.
- Maybe what connects polysemy and vagueness is related to the evolution of languages as means for communication.
  - More of this on day 4.

Contextualism: Background

- An early contextualist approach to vagueness is explored in Kamp (1981).
- Kamp originally defended supervaluationism (Kamp 1975)
- He became dissatisfied with the falsity of Tolerance
  - (the truth of $\neg\forall x\forall y((P(x) \wedge x \sim_P y) \rightarrow P(y)))$ implies $\exists x\exists y((P(x) \wedge x \sim_P y) \wedge \neg P(y))$
- Other contextualist approaches include: (Kamp 1981; Raffman 1996, 2000, 1996, 2000; Rayo 2008).

Basic idea

As we reason along the sorites series, there are changes in context that affect e.g., the meaning of an expression.

- Whenever we approach boundaries, context shift make them slip away further from our grasp.

The connection between context-sensitivity and vagueness I

An initial worry for a contextualist account:

- Vague terms in specific contexts are still vague
  - *Tall* (in the context of basketball players) is still vague

The standard response:

- Context switches are dynamic (e.g. Kamp 1981; Raffman 1996, 2000)
  - Context shifts can be evoked by: Seeing new stimuli, examining it more closely, considering similar cases
- Context switches are somewhat arbitrary (e.g. Raffman 1996, 2000; Rayo 2008)
  - Different agents switch judgements in different places on a forced march
  - The same agent can show difference when being march up or down a sorites series.

The connection between context-sensitivity and vagueness II

So vagueness is characterised more in terms of how we slip between contexts which subtly changes the meanings of expressions and so makes them hard to track.

Kamp's Contextualism

- One of the original proposals
- An oldie but a goody
- Influential in much other work

Kamp's Contextualism: Basic idea

Restriction on universal sentences: that means that it can be false without the equivalent existential sentence being true.

- $\neg\forall x\phi \rightarrow \psi$ can sometimes be true when $\exists x\phi \wedge \neg\phi$ is false
- Interpretations are dynamically updated in context
- the falsity of a universal can also occur when there is no *coherent context* in which all of its instances are true.

Kamp's Contextualism II

Supervaluationism

- A partial model
- Many classical model to complete the partial interpretations

Kamp's contextualism

- One model, many contexts
- Interpretations can be extended in context to cover cases in the gap.

Kamp's Contextualism III

- $\mathcal{I}_k$ is Kamp's interpretataion function
- U is the domain
- $B(c)$ is the set of background assumptions in context c
- $a \sim b$ means that a is tolerantly similar to b

$$\mathcal{I}_k(P(x_i))(c) = 1 \text{ iff } \exists a \in U(a \sim \mathcal{I}_k(x_i) \wedge P(a) \in B(c))$$

In words: x_i is in the extension of P at c iff

- There is an object a which is P, by background assumption, and which is tolerantly similar to x_i.

Kamp's Contextualism IV

Contexts are dynamic:

- the acceptance of a statement as true modifies the context
- this also adds this statement to the background assumptions

E.g., conditionals

$$\mathcal{I}_k(\phi \rightarrow \psi)(c) = 1 \text{ iff } \mathcal{I}_k(\phi)(c) = 0 \text{ or } \mathcal{I}_k(\psi)(mod(c, \phi)) = 1$$

In words: Standard material conditional semantics, but true when the consequent is true after c is updated with the antecedent.

This means that progressing down a Sorites series:

- the acceptance of each new item as P is based on the contextual assumptions and similarity to the previous member
- modifies the context so as to license the next move in the series (because $B(c)$ has been modified).

Incoherent Contexts and tolerance

Small extensions of contexts license extra small steps along a sorites series:

- This can make a context incoherent (if ϕ and $\neg\phi$ are both true in c)

Tolerance therefore holds, but not necessarily in a coherent context

- Instances of $\forall x \forall y((P(x) \wedge x \sim_P y) \rightarrow P(y))$ might be true at a context
- But if the context is incoherent, the universal statement is false (by definition)

$$\mathcal{I}_k(\forall x.\phi)(c) = 1 \text{ iff } \begin{array}{ll} (i) & mod(c, \forall x.\phi) \in Coh, \text{ and,} \\ (ii) & \text{for all } a \in U,\ \mathcal{I}(\phi_{x:=a})(c) = 1 \end{array}$$

In words: Standard semantics for a universal quantifier with the added condition (i):

- Modifying the context c with the proposition $\forall x.\phi$ leaves c coherent.

Incoherent Contexts and tolerance II

The semantics for the existential is more standard:

$$\mathcal{I}_k(\exists x.\phi)(c) = 1 \text{ iff for some } a \in U,\ \mathcal{I}(\phi_{x:=a})(c) = 1$$

This means that $\neg\forall x\phi \nvDash_k \exists\neg\phi$. Why?

- Updating c with a universal expression can make c incoherent and the universal false
  - I.e. Evaluating a tolerance universal forces you down the sorites march
- If the incoherence makes the universal false, there need not be any sharp boundary to make the existential true

Therefore the tolerance premise does not entail the truth of a sharp boundary proposition.

$$\neg\forall x\forall y((P(x) \wedge x \sim_P y) \rightarrow P(y)) \nvDash_k \exists x\exists y((P(x) \wedge x \sim_P y) \wedge \neg P(y))$$

thus remedying the problem Kamp highlighted with supervaluationism.

Summary

- Every tolerance conditional can be taken as true
- The universal tolerance statement is false, because it is not true in any coherent context
- This does not mean that the existential *sharp-boundary* statement is true.

Kamp's Contextualism and Higher-Order Vagueness

A Higher-Order Vagueness problems (that Kamp acknowledges)

- A sharp cut-off between coherent and incoherent contexts.

The impact of contextualism

Manifestations of related ideas are prevalent.

- the psychologically described contextualism of Raffman (1994, 1996),
- the contextualism of Tappenden (1993), van Deemter (1995) and Soames (1999),
- the dynamic semantics-based approaches of Barker (2002) and Lassiter (2011) (more on this tomorrow),
- in the similarity sensitivity of the Tolerant Classical Strict (TCS) approach (not time for this in this course, sadly)

Epistemicism: Change nothing

A common assumption so far

- Vagueness (esp. the sorites paradox) indicates a problem with CFOL and/or the nature of meaning representations

This led to e.g.,

- Multiple truth-values (fuzzy logical approaches)
- Paracompleteness (supervaluationism)
- Paraconsistency (subvaluationism)
- Linking semantics of $\forall$ to contextual update (contextualism)

Epistemicism is a essentially a backlash against this view:

- Vagueness and the sorites do not indicate a problem with CFOL
- Or a problem with the nature of classical meaning representations

Epistemicism: Core idea

Vagueness and the sorites indicate that we have imperfect knowledge of what our sharp-boundaried expressions mean

Epistemicism: Relocation, relocation, relocation

Essentially, epistemicism relocates the problem

- Vagueness is an epistemic phenomenon, not a semantic phenomenon

Original philosophical proponents:

- First proposed by Sorensen (1988) (updated in Sorensen (2001))
- Mostly attributed to Williamson (1992, 1994)

There are some differences

- Sorensen: We could never know where sharp boundaries are
- Williamson: We lack sufficient information and processing power to work out where they are.
    - We'll focus on Williamson's more influential version.

Epistemicism: Key properties

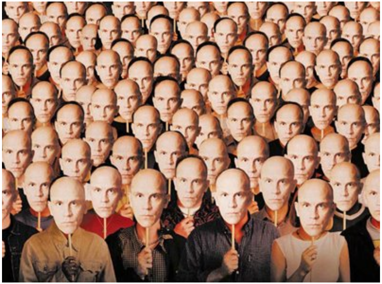
Some key properties of Epistemicism:

- No revision to semantics based on classical logic is necessary.
- The sorites argument is valid, but unsound because the Tolerance premise is straightforwardly false
- The tolerance premise is false because one of its instantiations is false (all the others can be true).
- Vagueness is a form of ignorance about where this sharp cut-off point is/which tolerance conditional is the false one.

Williamson's epistemicism: margins for error

A metaphor with vision:

- How many John Malkoviches are there in the photo?

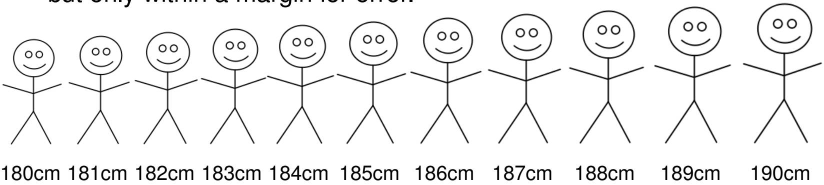


- Say that you form a belief by looking at the photo (without explicitly counting) and get the answer right
  - This true belief doesn't count as *knowing*.
- Knowledge, for Williamson, must support counterfactuals.
  - You believe 105, and you were right, but had there been 104 or 106, would your belief have been different?
  - In other words, do your belief forming abilities track the world properly, or do they only work within a particular *margin for error*

Williamson's epistemicism: margins for error

Vagueness works the same way:

- The relevant information we have:
  - How is the predicate used (meaning supervenes on use)
  - Contextual shifts - how is the predicate being used in this particular context
- This is enough information to know the boundaries of vague expression, but only within a margin for error.



- Epistemicism: There is a last tall person
- We can know very roughly where the last tall person is, but only within a margin of error (of say, ± 2.5cm)
- Even if we guess right, at, say, 187cm, if the boundary for *tall* had been 1cm different, would we have guess differently?

Criticisms of epistemicism I

- Most responses are ones of unpalatability

In Williamson's own words, "[a]lthough meaning may supervene on use, there is no algorithm for calculating the former from the latter"; again, "meaning may supervene on use in an unsurveyably chaotic way" [pp. 206,209]. That is, there is no hint in the epistemic theory as to where meanings do come from. To those who view the study of language as part of (or at least closely connected to) the study of human psychology and sociology, this consequence of epistemicism tends to come across as a reductio ad absurdum of the theory. (Lassiter 2011, p. 128)

Criticisms of epistemicism I

- Also reactions that re-emphasise the core intuitions about vagueness

the idea that if points *x* and *y* are very close in respect of colour, then the sentences ‘Point *x* is red’ and ‘Point *y* is red’ should be very close in respect of truth, is violated (Smith 2008, p. 177)

And similarly:

there should not be two consecutive and highly similar items *x* and *y* such that one ought to judge *x P* and one ought to judge *y* not *P*. There should be no ‘normative’ jolt in that sense. (Egré 2011, p. 74)

- Also reactions to Williamson’s epistemology (outside of our scope)

Degree Semantic accounts of vagueness

- Roots in Bartsch and Vennemann (1972); Cresswell (1977); Bierwisch (1989) (amongst others)
- Focus on the proposals of Fara (2000) and Kennedy (2007).
  - Both are epistemicist positions that try to make the jolt problem less alarming.

Degree Semantic accounts of vagueness: Basic idea

Add an extra semantic type d (for degree) to the familiar e and t.

- Gradable adjectives are typed as $\langle e, d\rangle$.
  - For an object, they return a degree on some relevant scale.
  - E.g., *tall* is a function from individuals to degrees of height.
- What we do not know is exactly which degree of height is the last/first degree for *tall*

Degree Semantic accounts of vagueness II

Suppose a degree based interpretation function $\mathcal{I}_d$:

$$\mathcal{I}_d(Tall_{\langle e,d\rangle}(a_{\langle e\rangle}))_{\langle d\rangle} = d$$

- This expresses that *a* is tall to degree *d*.

Bare-uses from comparative uses

Degree based approaches very successfully capture comparative constructions.

- Expressions such as ‘more’ or morphemes such as ‘-er’ are interpreted as functions to inequalities over degrees

$$\lambda P.\lambda y.\lambda x.F(x) > F(y)$$

- So *a* is taller than *b* ($Tall(a) > Tall(b)$) is true iff the degree of height of *a* is greater than that of *b*.

Null morpheme *pos* for ‘positive’ uses of gradable adjectives:

$$\lambda F.\lambda x.F(x) \geq \mathbf{X}_F$$

- $\mathbf{X}_F$: *d* on a scale determined by the adjective substituted for *F*

$$[[\text{Alex is tall}]] = Tall(a) \geq \mathbf{X}_{Tall}$$

Bare-uses from comparative uses

$\mathbf{X}_F$ should be context sensitive (Fara 2000; Kennedy 2007)

- E.g., if $\mathbf{X}_F$ is interpreted as *the average degree to which entities in the relevant class are F*, then the 'cut off' point for *F* will remain fixed given the contingent facts about entities which are in the extension of *F*.
  - I.e., *tall for a person* would equal *tall for a mountain*
- But, does not explain why vague predicates admit of borderline cases.

Two solutions to this problem.

- Interest relativity (Fara 2000)
- Domain of discourse restriction (Kennedy 2007).

Interest relativity (Fara 2000)

Fara (2000) proposes three modifications to the meaning of *pos*:

(i) an inclusion of a comparison class property P

(ii) $NORM : \langle\langle e, d\rangle, \langle\langle e, t\rangle, d\rangle\rangle$: a function from a measure function to a function from properties to degrees

(iii) $!>$ a 'significantly greater than' inequality (dependent on the agent's interests)

$$[pos]_{\text{fara}} = \lambda F.\lambda P.\lambda x.(F(x) \mathbin{!>} (NORM(F)(P)))$$

Given that the relevant property is *BB* (*being a basketball player*) '*a* is tall (for a basketball player)' as the following:

$$[[\text{Alex is tall (for a basketball player)}]] = Tall(a) \mathbin{!>} (NORM(Tall)(BB))$$

- I.e., Alex has a significantly greater degree of height that the normal degree of height for a basketball player

Domain of Discourse restriction (Kennedy 2007)

- Kennedy is concerned about interest relativity
  - Can ‘Everest is tall for a mountain’ ever be false no matter what your interests? (Stanley 2003, p. 278)
- Replace interest relativity with utterance context relativity

A context sensitive function, **s**, from measure functions to degrees.

- **s**(F) = the degree of *F*-ness in the context of utterance
  - selects a degree of *F*-ness, above which an object ‘stands out’ as *F*

$$[pos]_{\text{kennedy}} = \lambda F.\lambda x.(F(x) \geq \mathbf{s}(F))$$

Comparison classes are functionally composed with adjectives with a . combinator (Heim and Kratzer 1998)

- E.g. If $BB : \langle e, t\rangle$ and $Tall : \langle e, d\rangle$, then $BB.Tall : \langle e, d\rangle$
- Domain of *BB*.*Tall* restricted to basketball players.

[[Alex is tall (for a basketball player)]] $= (Bb(a).Tall(a)) \geq \mathbf{s}(Bb.Tall)$

Comparing degree semantic epistemicism with Williamson's epstemicism

- Williamson: We are ignorant as to the exact meanings of vague expressions
- Degree semantics: We know exactly what vague expressions mean, but we are ignorant about
  - (a) What the Norm for the comparison class is (Fara 2000)
  - (b) What the contextually determined standard is (Kennedy 2007)

Problems that persist:

- There is still a *jolt* ($a \sim_P b$, but $P(a) \wedge \neg P(b)$)

Some details not filled in:

- How do we characterize uncertainty about contextual standards?
- How do we characterize reasoning about these contextual standards?

Both of these details addressed tomorrow:

- Probabilistic accounts of vagueness

Summary

	Fuzzy Logic	Superval	Subval	Kamp's Contextualism	Epistemicism/ Degrees
No gaps	No	No	Yes	Yes	Yes
No gluts	Yes	Yes	No	Yes	Yes
Excluded Middle	No	Yes	Yes	Yes	Yes
Non-Contradiction	No	Yes	Yes	Yes	Yes
Modus Ponens	Yes	Yes	No	Yes	Yes
Quantifier Duality	Yes	Yes	Yes	No	Yes

Take-home messages for day 2

- S’valuationism retains a lot of the properties of classical logic...
- ... but there are bullets to bite
  - Paracompleteness/paraconsistancy
  - Problems with tolerance
- There is something to say for S’valuationist logic as a logic for statements formed with polysemous expressions
  - It is much less clear that vagueness is a kind of polysemy though
- Contextualism à la Kamp also stays close to CL
  - Still work to be done on the formal implications
  - Problems with higher-order vagueness
- Degree Semantics with Epistemicism
  - Makes Epistemicism more palatable
  - Remaining ‘jolt’ problem
  - Some details about reasoning bout contextual standards needed

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