Vagueness & Polysemy 1

Vagueness as a problem for first-order classical logic

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Winter School on Semantics in Barcelona

January, 2018

Main goals for the course

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Days 1-3: Formal accounts of vagueness

- To understand why vagueness is a challenge for most formal approaches to semantics and what the main responses to this challenge have been.
- An awareness of how these responses do/do not connect vagueness to polysemy.

Day 4: The origins of vagueness and polysemy

- An introduction to some accounts of the origins of vagueness and polysemy (both arise from learning/communication)

Main questions for the course

Main Questions

- Why is vagueness a challenge for a formal semantics for natural languages?
  - Day 1: Background
    - Properties of Classical First Order Logic (CFOL) & connection to modern semantic theory
    - Diagnosing Vagueness
- What are the main reactions to this challenge?
  - Days 1-3: Responses to vagueness
    - fuzziness (day 1)
    - S'valuationism & polysemy, contextualism, degrees (day 2)
    - probabilistic approaches (day 3)
- How does vagueness connect to polysemy?
  - Part of Day 2: Is vagueness a kind of polysemy?
  - Day 4: The common origins of vagueness and polysemy?

Plan for Day 1

1. Background
2. Vagueness as a challenge for classical logic
   - Properties of CFOL
   - Diagnosing Vagueness in Natural Languages
3. Overview of some further issues
   - Absolute versus Relative Scalar Adjectives
   - Higher-Order Vagueness
4. Semantic treatments of vagueness
   - Traditional fuzzy approaches
   - An intensional fuzzy approach

Vagueness: an everyday term or a technical term?

Austin proposes a non-exhaustive list of features of vague descriptions:
“It might be

(a) a rough description, conveying only a ‘rough idea’ of the thing to be described; or
(b) ambiguous at certain points, so that the description would fit, might be taken to mean, either this or that; or
(c) imprecise, not precisely specifying the features of the thing described; or
(d) not very detailed; or
(e) couched in general terms that would cover a lot of rather different cases; or
(f) not very accurate; or perhaps also
(g) not very full or complete...

...there is not just one way of being vague, or one way of not being vague, viz. being precise.” (Austin 1962, pp. 125-6)

- But in philosophy and linguistics, *vagueness* is usually used in a more technical sense.
  - But there isn’t total consensus on the technical sense either...

Vagueness as a technical term?

- Usually, if a predicate, P, is vague, then it has at least one of the following properties:
  1. Borderline cases
     - ★ There is at least one entity, a_i, such that P applies equally well/as fully to a_i as not-P.
     - ★ This colour is clearly red
     - ★ This colour is neither clearly red nor clearly not red
     - ★ This colour is clearly not red
  2. Blurred boundaries
     - ★ There is (seemingly) no clear/sharp cut-off point between the extensions of P and not-P.
     - ★ Red grades into orange which grades into yellow
  3. Sorites susceptibility/tolerance
     - ★ small differences in objects cannot make a P object a not-P object
     - ★ We are embarrassed to locate any sharp boundaries in a graded series

Vagueness across grammatical categories

- Much work on vagueness in linguistics has focussed on (gradable) adjectives:
  - *tall*, *short*, *bald*, *red*
- But the three properties associated with vagueness (borderline cases, blurred boundaries, tolerance) can be found in many parts of speech
  - CNs: *cat* vs. *kitten*; *chair* vs. *stool*
  - Verbs: *walk* vs. *jog* vs. *run*
  - PPs: *under the table* vs. *next to the table*
  - Adverbs: *quickly* vs. *slowly*, *quite*, *very*
  - Determiners: *many*, *much*, *little*, *few*

Vagueness across philosophical categories

There is debate regarding what the philosophical nature of vagueness is:

Metaphysical

- The root source of vagueness is the world: objects themselves are vague.
  - Rarer view, but see Smith (2008)
  - Some puzzles on identity conditions (Evans 1978)

Semantic

- Vagueness derives from meanings/meaning representations.
  - Majority viewpoint up until late 1990s.
  - Still widely defended.

Epistemic/doxastic

- Vagueness derives from our relationship (e.g. knowledge/belief) to meanings.
  - Increasingly influential from late 90s onwards.
  - Incorporated into other formal semantic accounts (e.g., degree semantics Kennedy 2007)

A puzzle for metaphysical vagueness

Evans (1978) argues that the notion of vague objects is incoherent:

- If objects are vague, then, for some objects a, b, it should be indeterminate whether $a = b$
- An indeterminacy operator $\triangledown$

$\triangledown(a = b)$	Assumption
$(\lambda x[\triangledown(a = x)])(b)$	lambda abstraction
$\neg \triangledown (a = a)$	Assumption
$\neg(\lambda x[\triangledown(a = x)])(a)$	lambda abstraction
$a \neq b$	Leibnitz's law

Another puzzle for metaphysical vagueness

Metaphysical vagueness is attractive because it affords a simple view of the relationship between language and the world.

- Predicates (straightforwardly) refer to properties in the world.
- The properties in the world are vague.

Arguably seems attractive for something like *red*

- *red* straightforwardly picks out the vague property RED.

But it is quite hard to see how this applies in all cases.

- *under the table* straightforwardly picks out the vague property UNDER-THE-TABLE-NESS?
- many straightforwardly picks out the vague property MANY-NESS?

From here on, we'll mostly stick to the semantic and epistemic conceptions.

CFOL

Classical Logic underpins most work in formal (model-theoretic) semantics.[1]

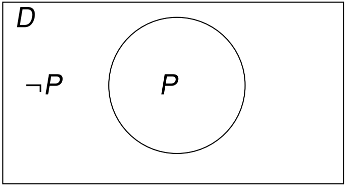
- Review: Some properties of Classical First-Order Logic (CFOL)
- Overview: relevance to more complex and higher-order semantics
- Why are the diagnostic properties of vagueness a challenge?

[1] Intuitionistic logics have their own challenges when it comes to vagueness.

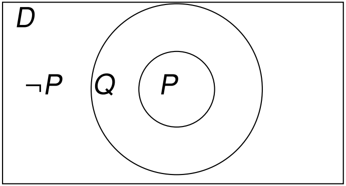
No gaps

Truth-value gaps arise when predicates do not form complete partitions over the domain.

P forms a total partition (no gaps)



P forms a partial partition (gaps)
Assume that $Q-P$ is neither P nor not-P



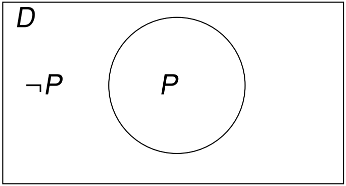
No Gaps:

For all $\mathcal{I}$, all $a \in D$ and all predicates P,
$\mathcal{I}(P(a)) = 1$ or $\mathcal{I}(P(a)) = 0$.

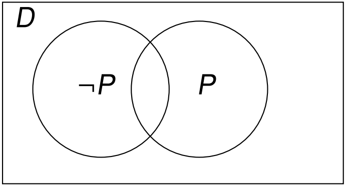
No Gluts

Truth-value gluts arise when predicates do not form proper functions over the domain such that they specify more than one output for some entities in the domain.

P is a proper function on D (no gluts)



P is not a proper function on D i.e. for some $a \in D$, more than one value for $P(a)$ (gluts)



No Gluts:

For all $\mathcal{I}$, all $a \in D$ and all predicates P,
It is not the case that $\mathcal{I}(P(a)) = 1$ and $\mathcal{I}(P(a)) = 0$.

Bivalence

No Gaps and *No Gluts* are usually taken together under the single metasemantic *principle of bivalence*:

Bivalence

For all $\mathcal{I}$, all $a \in D$ and all predicates P,
exclusively, either $\mathcal{I}(P(a)) = 1$ or $\mathcal{I}(P(a)) = 0$.

- As we'll see later though, sometimes *no gaps* and *no gluts* have to be kept apart
  - Bivalence can fail due to gappiness or glutiness

Contradiction with Explosion

In CFOL, contradictions allow one to infer any proposition (explosion)

An intuitive explanation:

- Material conditionals are true if their antecedents are true
- So $(\phi \land \neg\phi) \rightarrow \psi$ is true for any ψ

In terms of proof trees:

- An argument is valid if the proof tree of its premises and the negation of its conclusion closes.
- If the premise is a contradiction, every branch of the tree closes, so the argument is valid no matter what the conclusion is.

Contradiction with Explosion:

For all formulas ϕ, ψ,
$\{\phi, \neg\phi\} \vDash \psi$

Law of the Excluded Middle (LEM)

No Gaps was a metasemantic principle (stated in terms of the semantic properties of object language predicates).

The principle of the excluded middle is the parallel object language statement (and a theorem of CFOL).

- Nb. As we'll see, one can hold without the other.

Excluded Middle:

For all predicates P,
$\vDash \forall x(P(x) \vee \neg P(x))$

In other words, all interpretations satisfy $P(a) \vee \neg P(a)$, for all $a \in D$.

Non-Contradiction

No Gluts was a metasemantic principle (stated in terms of the semantic properties of object language predicates).

The principle of non-contradiction is the parallel object language statement (and a theorem of CFOL).

- Nb. Again, as we'll see, one can hold without the other.

Non-contradiction:

For all predicates P,
$\vDash \forall x_1 \neg(P(x) \wedge \neg P(x))$

In other words, all interpretations satisfy $\neg(P(a) \wedge \neg P(a))$, for all $a \in D$.

Modus (ponendo) ponens

Modus ponens is valid in FOL.

- Surprisingly, vagueness can lead to doubting even this cornerstone of reasoning

Modus Ponens:

For all formulas ϕ, ψ,
$\{\phi \rightarrow \psi, \phi\} \vDash \psi$

Duality of $\exists$ and $\forall$

The quantifiers $\exists$ and $\forall$ are duals. For example:

- Some ducks quack iff not all ducks don't quack.
- All ducks swim iff no ducks don't swim.

Quantifier duality:

For all formulas ϕ

$\exists x\phi \equiv \neg\forall\neg\phi$

and

$\forall x\phi \equiv \neg\exists\neg\phi$

Extensions to CFOL in linguistics

There have been many developments in semantics that require a richer or else more complex logic that CFOL. For example:

- Most formal semantic analyses assume a higher-order logic (minimally, quantification over predicates)
- Richer varieties of types (worlds, events, degrees etc.)
- Two dimensional interpretations (contexts)
- Mereologically structured domains

Extensions to CFOL in linguistics

However, many properties of CFOL are retained:

- Total interpretation functions, with $\{0, 1\}$ being the only truth values.
- Constituents are still assigned sets (may consist of different sorts of objects than in CFOL, but still sharp).
- Negation is usually either a propositional truth-reversing operator (ex. as in Chierchia and McConnell-Ginet (2000), Heim and Kratzer (1998) etc.) or as a more general complement operator (ex. Keenan and Faltz (1985), Winter (2001)).
- Entailment in NLs has the same properties as in CFOL (Heim and Kratzer (1998), Chierchia and McConnell-Ginet (2000) a.m.o).

E.g., Bivalence, LEM, Non-contradiction, Quantifier duality, Modus Ponens

- One cannot, arguably, have all of these and account for vagueness

Borderline cases

Say the average height is 175cm. Alex is 177cm in height. Is Alex tall?

- It seems that Alex is neither clearly tall nor clearly not tall .
- In fact, it seems natural to say both:
  1. Alex is neither tall nor not tall
  2. Alex is both tall and not tall.

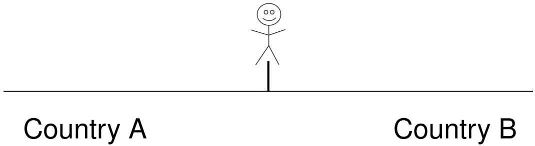
Furthermore, there have been experimental studies that indicate that "both" and/or "neither" answers seem to be favoured by NL speakers (Ripley 2011; Alxatib and Pelletier 2011; Serchuk et al. 2011; Egré et al. 2013).

- However, these are prima facie counter examples to excluded middle and non-contradiction:
  1. $\neg(T(a) \vee \neg T(a))$ (denies excluded middle)
  2. $T(a) \wedge \neg T(a)$ (denies non-contradiction)
- Or, at least, indicate either gaps or gluts

Insufficiency of borderline cases

A criticism of a borderline-case conception of vagueness is that it seems insufficient for vagueness.

- Suppose that a person is sitting on a fence that marks the border between two countries, A and B.
- Even if the border is precisely defined, the person is not clearly in country A or country B



Blurred/fuzzy boundaries

An arguably better analysis is a continuum.

- Some things are clearly *P*
- Some things are clearly not-*P*
- There is a continuum of *P*-ness that spans between the clear cases.

The effect on semantics turns on how we characterise the continuum between ‘clear cases’ versus ‘non-clear cases’

1. Introduce a continuum of truth-values
   - Lose bivalence, possibly lose modus ponens, possibly lose transitivity
2. Stick with two values, but export the issue to beliefs/knowledge about (classical) truth conditions.
   - Raises the question what rues reasoning in such cases follow (probability?)

Sorites susceptibility and tolerance

Sorites arguments (from ancient Greek for *heaper*) first discussed by Eubulides of Miletus.

- If many grains of sand form a heap, removing a single grain cannot turn the heap into a non-heap. Removing one grain therefore, leaves a heap. Removing another grain therefore leaves a heap, ..., one/no grains of sand make a heap.

Sorites susceptibility and tolerance

The form of the argument as usually adopted is either:

- the long form (many conditional premises)
- the short form with a universal premise.

Assume a relation $\sim_P$ such that if $a_1 \sim_P a_2$, then a_1 differs from a_2 in some marginal way with respect to P. Also assume some arbitrarily long series $a_1, ..., a_n$ such that $a_i \sim_P a_{i+1}$

The long sorites

(P1) $P(a_1)$

(P2) $\neg P(a_n)$

(P3) $P(a_1) \wedge a_1 \sim_P a_2) \rightarrow P(a_2)$ (tolerance conditional)

(P4) $P(a_2) \wedge a_2 \sim_P a_3) \rightarrow P(a_3)$

(P5) .....

(P6) $P(a_{n-1}) \wedge a_{n-1} \sim_P a_n) \rightarrow P(a_n)$

(C) $P(a_n) \wedge \neg P(a_n)$

The short sorites

(P1) $P(a_1)$

(P2) $\neg P(a_n)$

(P3) $\forall x \forall y [P(x) \wedge x \sim_P y) \rightarrow P(y)]$ (sorites premise/tolerance principle)

(C) $P(a_n) \wedge \neg P(a_n)$

Why is the sorites really a problem

(P1) $P(a_1)$ (assumption)

(P2) $\neg P(a_n)$ (assumption)

(P3) $\forall x \forall y[P(x) \wedge x \sim_P y) \rightarrow P(y)]$ (sorites premise/tolerance)

(C) $P(a_n) \wedge \neg P(a_n)$

The trouble:

- The argument is classically valid.
- The two assumptions are clearly true.
- So, arguably, either the tolerance principle is false, or we need a non-classical notion of entailment.

Tolerance principle is false:

- $\neg(\forall x \forall y[P(x) \wedge x \sim_P y) \rightarrow P(y)]) \equiv \exists x \exists y[P(x) \wedge x \sim_P y \wedge \neg P(y)]$
- But that means that P has sharp boundaries, contrary to the characteristic properties of vagueness.

A non-classical notion of entailment

- What should we drop/amend?
  - Lots of options, some to be considered over the coming days

Absolute and relative scalar adjectives

Most adjectives we have discussed so far = *relative scalar adjectives*

- *tall*, *fast*, *heavy*

But there are also *absolute scalar adjectives*

- *bent*, *empty*, *wet*

Can be distinguished with modifiers e.g.:

(1) #Almost/absolutely/completely tall/fast/heavy

(2) Almost/absolutely/completely bent/empty/wet

Tolerance principles and relative scalar adjectives

Relative scalar adjectives support positive and negative tolerance principles

- For some small measure of height *x*, if someone is x shorter than a tall person, then they are both tall.
- For some small measure of height *x*, if someone is x tall than a person who is not tall, then neither of them are tall.

Tolerance principles and absolute scalar adjectives

Absolute scalar adjectives only support positive tolerance principles Égré and Bonnay (2010); Burnett (2012, 2014)

- For some very small amount of liquid x, a glass that has x more liquid than an empty glass is also empty.
- #For some very small amount of liquid x, a glass that has x less liquid than a non-empty glass is not empty.

- We can be tolerant with positive forms: not completely wet/dry/straight/bent things can still be wet/dry/straight/bent.
- We cannot be tolerant with negative forms: there is a point at which not completely wet/dry/straight/bent things become completely wet/dry/straight/bent.

Varieties of Higher-Order Vagueness

As we will see, even if the problem of vagueness can be solved at the first order level, it has a habit of cropping up again 'higher' up.

But, to even talk about higher-order vagueness is a challenge:

- The term is used in different ways, so it isn't always clear what is meant.
- There is some question as to whether it really exists (Wright 2009)

There are at least three conceptions (not exclusive of each other)

1. Lexical Higher-Order Vagueness
   - Mostly concerns the semantics of adverbial modifiers e.g., absolutely, really, very
2. Formal Higher-Order Vagueness
   - Mostly concerns formal *definitely operators* and the logic thereof
3. Metasemantic Higher-Order Vagueness
   - Mostly concerns entitlement or justification to say/believe something using first-order vague expressions or concepts.

Lexical Higher-Order Vagueness

The semantic analysis of natural language expressions such as *definitely*, *truly*

- Compositional analyses for e.g., *really very tall*, *definitely very tall*, *truly tall* etc.
  - A relatively bounded enterprise
  - Open question: should expressions like the ones above come out as vague as e.g., *tall*

Formal Higher-Order Vagueness I

A tension

If a semantic theory accommodates vagueness by discriminating one or more areas in the extension of a predicate beside those for which the predicate is true *simpliciter* or false *simpliciter*, what can be said about the boundary between the positive (or negative) extension and the intermediate areas?

- positive extension: $\mathcal{I}(P^+)$
- negative extension: $\mathcal{I}(P^-)$
- intermediate: $\mathcal{I}(P^\pm)$
  - Is it ok that there are sharp cut-offs between $\mathcal{I}(P^+)$ and $\mathcal{I}(P^\pm)$ and between $\mathcal{I}(P^\pm)$ and $\mathcal{I}(P^-)$?

Formal HOV is usually framed in terms of a formal Δ (definiteness) operator.

Formal Higher-Order Vagueness II

Suppose:

- $\mathcal{I}(P) = \{a_1, a_2, a_3\}$
- $\mathcal{I}(\neg P) = \{a_4, a_5, a_6\}$

Can the sharp cut-off between P and $\neg P$ be assuaged by thinking about what is definitely P and definitely not-P? Suppose:

- $\Delta(P)$ is $\{a_1, a_2\}$
- $\Delta(\neg P)$ is $\{a_5, a_6\}$
    - No sharp cut-off between what is definitely P and definitely not-P.

BUT, now a sharp cut-off between $\Delta(P) = \{a_1, a_2\}$ and $\neg\Delta(P) = \{a_3, a_4, a_5, a_6\}$.

Iteration problems

Asking what is definitely definitely P ($\Delta(\Delta(P))$) may remove the sharp cut-off at the second order, but introduces one at the third (between $\Delta(\Delta(P))$ and $\neg\Delta(\Delta(P))$), and so on.

Metasemantic Higher-Order Vagueness I

Not normally referred to as HOV, but structural similarities.

The problem

No matter what a semantic theory says about the extension of P (or the truth conditions of P), are we still left with a problem of under what conditions one would be justified/correct/entitled to assert/believe P? (see Wright (1975))

Suppose:

- a theory of vagueness results in a completely smooth transition between P and $\neg P$
  - e.g., in terms of degrees of truth or something else
- Question: Which point should one cease to use P in, for example a P-based sorites series?
  - Even if we get vague truth conditions right, does a related problem arise for e.g., correct or justified use-conditions?

Metasemantic Higher-Order Vagueness II

Two options seem to be available:

(A) Develop a semantics which has sharp boundaries.

- Allows an easy mapping between when it is right to apply *P* (or not, or hedge)
- But is it a satisfactory theory of vagueness?

(B) Develop a semantic theory of vagueness that provides graded denotations of predicates.

- Arguably a better account of the truth conditions of vague predicate
- But, cannot straightforwardly map semantics into, say, a theory of correct assertion.

If (B), there are two further options:

(B1) pick a point on the graded scale as the last point at which one can correctly assert *P*

- Back to sharp boundaries...

(B2) endorse a graded view of e.g. justified assertion

- Problem can be shifted 'upwards'
- E.g., when we are *truly* justified in applying *P*?

Summary

The key properties of CFOL

- Bivalence (no gaps and no gluts)
- Excluded Middle
- Non-Contradiction
- Modus Ponens
- Quantifier Duality

Diagnosing Vagueness

- Borderline cases
- Blurred boundaries
- Sorites susceptibility

The problem

- Vagueness means that, prima facie, we can't accommodate the properties of vagueness and keep the properties of CFOL

Outline

Review some of the main semantic approaches to vagueness.

- Fuzzy logical approaches (today)
- S'valuationism (tomorrow)
- Contextualism (tomorrow)
- Epistemicism and degree semantics (tomorrow)
- Probabilistic approaches (Thursday)
- Lacuna: Tolerant, Classical, Strict (not enough time)

Grouped in semi-chronological order
Each make different adjustments to the basic CFOL set-up (if any)
Review some of the challenges for each

Background to fuzzy-logical approaches to vagueness

- Grew out of early investigations into multiple valued logic by Łukasiewicz (1922/1970)
- independently developed work into fuzzy sets (Zadeh 1965)
- development of a logic based on fuzzy sets (Goguen 1969)

Main idea

Bivalence is the problem.

- Two truth values are not enough to capture the blurred nature of vagueness.
- Classical sets are artificially sharp. We need fuzzy sets.

Fuzzy sets and fuzzy logic

Classical sets that form a total partition on the domain:

- for all $a \in D$, either $a \in A$ or $a \notin A$

Fuzzy sets replace categorical membership with membership to some degree in the range $[0, 1]$

- for all $a \in D$, $|a \in A| \in [0, 1]$

Fuzzy logic (based on fuzzy set theory)

- Replaces the two truth values of classical logic
- with an infinite number of degrees of truth in the range $[0, 1]$

If Alex is borderline tall, then the sentence *Alex is tall* will be true to some intermediate degree (e.g., 0.5).

Classical connectives

The standard classical connectives $\wedge$ and $\vee$ can be defined using *min* and *max* functions:

Interpretation of classical connectives

For an interpretation function for classical values $\mathcal{I}_c$, and for all propositions ϕ, ψ:

1. $\mathcal{I}_c(\phi) \in \{0, 1\}$
2. $\mathcal{I}_c(\neg\phi) = 1 - \mathcal{I}_f(\phi)$
3. $\mathcal{I}_c(\phi \wedge \psi) = min\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$
4. $\mathcal{I}_c(\phi \vee \psi) = max\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$

Which gives the standard truth tables:

ϕ	ψ	$\neg\phi$	$\phi \wedge \psi$	$\phi \vee \psi$
1	1	0	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	0

Fuzzy connectives

Fuzzy connectives can be defined the same way, except that the range of values $[0, 1]$ replaces the classical $\{0, 1\}$

Interpretation of Fuzzy Logical Propositions

For an interpretation function for fuzzy values $\mathcal{I}_f$, and for all propositions ϕ, ψ:

1. $\mathcal{I}_f(\phi) \in [0, 1]$
2. $\mathcal{I}_f(\neg\phi) = 1 - \mathcal{I}_f(\phi)$
3. $\mathcal{I}_f(\phi \wedge \psi) = min\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$
4. $\mathcal{I}_f(\phi \vee \psi) = max\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$
5. $\mathcal{I}_f(\phi \rightarrow \psi) = \begin{cases} 1 & \text{if } \mathcal{I}_f(\phi) \leq \mathcal{I}_f(\psi) \\ 1 - \mathcal{I}_f(\phi) + \mathcal{I}_f(\psi) & \text{if } \mathcal{I}_f(\phi) > \mathcal{I}_f(\psi) \end{cases}$

As a result:

- Fuzzy values of formulas are classical at the limit

(The definition for all, including the conditional are based on Łukasiewicz (1922/1970).)

Fuzzy conjunction: $\mathcal{I}_f(\phi \wedge \psi) = min\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$

Some prima intuitive ideas:

- If Alex, Billie, Charlie, and Drew are all equally borderline tall, then the following formulas should have the same value:
  - Alex is tall.
  - Alex and Billie are tall.
  - Alex, Billie and Charlie are tall.
  - Alex, Billie, Charlie and Drew are tall.
- If Alex is not as tall as Billie, then
  - $\mathcal{I}_f(Tall(a) \wedge Tall(b))$ should equal $\mathcal{I}_f(Tall(a))$

Exercise:

Suppose the following:

$$\mathcal{I}_f(\phi) = 0.9 \qquad \mathcal{I}_f(\psi) = 0.5 \qquad \mathcal{I}_f(\tau) = 0.1$$

What are the values for the following formulas?

$$\phi \wedge \psi \qquad \psi \wedge \tau \qquad (\tau \wedge \psi) \wedge \phi$$

Fuzzy Negation and Disjunction

$\mathcal{I}_f(\phi) \in [0, 1]$
$\mathcal{I}_f(\phi \vee \psi) = max\{\mathcal{I}_f(\phi), \mathcal{I}_f(\psi)\}$
Some prima intuitive ideas:

- If Alex and Billie are equally borderline tall, then the following formulas should have the same value:
  - Alex is tall.
  - Alex or Billie is tall.
- If Alex is not as tall as Billie, then
  - $\mathcal{I}_f(Tall(a) \vee Tall(b))$ should equal $\mathcal{I}_f(Tall(b))$
- The degree to which it is true that Alex is tall should equal the degree to which it is false that Alex is not tall
  - Degree of falsity is plausibly 1 – degree of truth

Excluded middle and non-contradiction

Recall that in CFOL, both of the following are theorems

$$\phi \vee \neg\phi \quad \text{(excluded middle)}$$
$$\neg(\phi \wedge \neg\phi) \quad \text{(non-contradiction)}$$

For fuzzy logic (as defined here), if 1 is the only designated value, then are either excluded middle or non-contradiction theorems?

- Excluded middle: Do all interpretations satisfy $P(a) \vee \neg P(a)$, for all $a \in D$?
  - No. If $0 < \mathcal{I}_f(P(a)) < 1$, then $\mathcal{I}_f(P(a) \vee \neg P(a)) < 1$.
  - In fact, if $\mathcal{I}_f(P(a)) = 0.5$, $\mathcal{I}_f(P(a) \vee \neg P(a)) = 0.5$.
- Non-contradiction: Do all interpretations satisfy $\neg(P(a) \wedge \neg P(a))$, for all $a \in D$?
  - No. If $0 < \mathcal{I}_f(P(a)) < 1$, then $\mathcal{I}_f(\neg(P(a) \wedge \neg P(a)) < 1$.
  - In fact, if $\mathcal{I}_f(P(a)) = 0.5$, $\mathcal{I}_f(\neg(P(a) \wedge \neg P(a)) = 0.5$.

Excluded middle and non-contradiction

We can try to rescue Excluded Middle and Non-Contradiction, however

- Instead of the only designated value being 1, designated values can be all values ≥ 0.5

See Smith (2008) for extensive discussion, but in brief:

- Formulas with values ≥ 0.5 are e.g., 'true enough'
- Classical theorems all come out as ≥ 0.5

Modus Ponens

In fact, not clear whether modus ponens is universally applicable.

- Depends on the notion of fuzzy validity...
- ...and on what definition we take for the conditional
  - Should the falsity of the conclusion not exceed the sum of falsity of the premises?
  - If so, modus ponens is valid (as are multiple applications of it)...
  - ...but single conjoined premise equivalents are not
    - ⋆ $\phi \wedge (\phi \rightarrow \psi) \wedge (\phi \rightarrow \tau) \vDash \tau$

Fuzzy logic and the sorites

Suppose

x	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}
$\mathcal{I}_f(P(x))$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

For any tolerance conditional $P(a_n) \rightarrow P(a_{n+1})$, we can apply the fuzzy conditional rule:

- $\mathcal{I}_f(\phi \rightarrow \psi) = \begin{cases} 1 & \text{if } \mathcal{I}_f(\phi) \leq \mathcal{I}_f(\psi) \\ 1 - \mathcal{I}_f(\phi) + \mathcal{I}_f(\psi) & \text{if } \mathcal{I}_f(\phi) > \mathcal{I}_f(\psi) \end{cases}$

Since, $\mathcal{I}_f(P(a_n)) > \mathcal{I}_f(P(a_{n+1}))$:

- $\mathcal{I}_f(P(a_n) \rightarrow P(a_{n+1})) = 0.9$
  - E.g. $1 - 1 + 0.9 = 0.9$, $1 - 0.9 + 0.8 = 0.9$, ..., $1 - 0.1 + 0 = 0.9$
- The conjunction of these premises will also evaluate to 0.9
- The conclusion $P(a_{11})$ evaluates to 0
- Is the argument valid?
  - No for a single premise: the degree of truth of the conclusion can be far less than the degree of truth of the conjunction of the premises
  - Yes for multi-premises: the degree of falsity of the conclusion does not exceed the sum of degrees of falsity of the premises

Advantages and disadvantages of fuzzy approaches

advantages

- A genuinely graded 'slide' down the slippery slope
- Some explanation for why borderline cases admit of denials of excluded middle and non-contradiction

disadvantages

- Losing excluded middle and non-contradiction is quite a lot to stomach
- Not clear where the single premise invalidity of the sorites is coming from
  - Why can we assert the conjunction of the premises but not the final consequent of the last conditional?
- Not altogether clear what degrees of truth are
- Metasemantic HOV: What is the highest fuzzy degree d such that if ϕ is evaluated at d, then ϕ is correctly assertable?
  - What about $d - \epsilon$?

Edgington's (1997) argument against fuzzy conjunction

Suppose that Alex is borderline tall

- $\mathcal{I}_f(\mathit{Tall}(a)) = 0.5$

Also suppose that Billie is marginally less tall than Alex, e.g.

- $\mathcal{I}_f(\mathit{Tall}(b)) = 0.4$

Shouldn't it be perfectly false to say that Billie is tall, but Alex is not?

Not with fuzzy conjunction

- $\mathcal{I}_f(\mathit{Tall}(b) \wedge \neg \mathit{Tall}(a)) = \mathit{min}(\{0.4, 1 - 0.5\} = \mathit{min}\{0.4, 0.5\} = 0.4$

An updated fuzzy approach (Alxatib et al. 2013)

Based partly on experiments mentioned earlier, Alxatib et al. (2013) embrace the idea that flat contradictions can be completely true.

- Intensional fuzzy connectives e.g., $\otimes$
- Calculated from basic fuzzy connectives + *intensional* elements: floor and ceiling values
- a scaling operation.

Floor values

The minimal possible value for the fuzzy conjunction. E.g., $\mathbf{f}(\phi \wedge \psi)$

Ceiling values

The maximal ceiling possible value for the fuzzy conjunction. E.g., $\mathbf{c}(\phi \wedge \psi)$

$$\mathcal{I}_f(\phi \otimes \psi) = \begin{cases} \mathcal{I}_f(\phi \wedge \psi) & \text{if } \mathbf{f}(\phi \wedge \psi) = \mathbf{c}(\phi \wedge \psi) \\ \dfrac{\mathcal{I}_f(\phi \wedge \psi) - \mathbf{f}(\phi \wedge \psi)}{\mathbf{c}(\phi \wedge \psi) - \mathbf{f}(\phi \wedge \psi)} & \text{otherwise} \end{cases}$$

An updated fuzzy approach (Alxatib et al. 2013)

$$\mathcal{I}_f(\phi \otimes \psi) = \begin{cases} \mathcal{I}_f(\phi \wedge \psi) & \text{if } \mathbf{f}(\phi \wedge \psi) = \mathbf{c}(\phi \wedge \psi) \\ \dfrac{\mathcal{I}_f(\phi \wedge \psi) - \mathbf{f}(\phi \wedge \psi)}{\mathbf{c}(\phi \wedge \psi) - \mathbf{f}(\phi \wedge \psi)} & \text{otherwise} \end{cases}$$

Flat contradictions come out completely true for borderline cases

Assuming that $\mathcal{I}_f(\phi) = \mathcal{I}_f(\neg\phi) = 0.5$

$$\begin{aligned} \mathcal{I}_f(\phi \wedge \neg\phi) &= 0.5 \\ \mathbf{f}(\phi \wedge \neg\phi) &= 0 \\ \mathbf{c}(\phi \wedge \neg\phi) &= 0.5 \\ \mathcal{I}_f(\phi \otimes \neg\phi) &= \frac{0.5 - 0}{0.5 - 0} = 1 \end{aligned}$$

Edgington's argument again

Suppose that Alex is borderline tall

- $\mathcal{I}_f(\mathit{Tall}(a)) = 0.5$

Also suppose that Billie is marginally less tall than Alex, e.g.

- $\mathcal{I}_f(\mathit{Tall}(b)) = 0.4$

Shouldn't it be perfectly false to say that Billie is tall, but Alex is not?

- Still not with an intensional fuzzy system
  - Floor$=0$: Heights are contingent, Alex could have been perfectly tall and Billie could have been perfectly short
  - Ceiling$=1$: Heights are contingent, Alex could have been perfectly short and Billie could have been perfectly tall

$$\begin{aligned}
\mathcal{I}_f(\mathit{Tall}(b) \wedge \neg \mathit{Tall}(a)) &= 0.5 \\
\mathbf{f}(\mathit{Tall}(b) \wedge \neg \mathit{Tall}(a)) &= 0 \\
\mathbf{c}(\mathit{Tall}(b) \wedge \neg \mathit{Tall}(a)) &= 1 \\
\mathcal{I}_f(\mathit{Tall}(b) \mathbin{\circledwedge} \neg \mathit{Tall}(a)) &= \frac{0.5 - 0}{1 - 0} = 0.5
\end{aligned}$$

Summary: Departures from CFOL

	Fuzzy Logic
No gaps	No
No gluts	Yes
Excluded Middle	No
Non-Contradiction	No
Modus Ponens	Yes
Quantifier Duality	Yes

Take-home messages for day 1

- Vagueness, if taken seriously, poses non-trivial problems for NL semantics as it is usually done
- Fuzzy approaches are a direct way to tackle vagueness
  - Radical departure from CFOL
  - Opens up some rather deep metaphysical questions

Thanks

A huge thanks to Heather Burnett. Much of today's material is based on a handbook article we have co-written.

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